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# UNSTEADY MELT FLOWS IN THE CYLINDRICAL CONTAINER

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*Leading topic of this article is description of unsteady melt flow Lorentz forces in the container with cylindrical shape. This melt is driven by rotating magnetic field. Numerical simulations of these unsteady flows were performed in commercial software Ansys Fluent (version 13). Taylor number of this flow is $10^6$. Lorentz forces computed in software Ansys Fluent were compared with results of analytic formula for Lorentz forces. Unsteady melt flow was described by contours of Lorentz forces and velocity field (azimuthal and axial).*

**Keywords:** MHD, Lorentz forces, magnetohydrodynamics

## Introduction

Magnetohydrodynamics (MHD for short) is a theory of interaction between magnetic field and moving, conducting fluids [2]. First mentions about MHD appeared in relation to astrophysics and geophysics. The interest in MHD was extended to industry later. Rotating magnetic field generates eddy flow in electric conductive melt. This effect is used to e.g. for non-contact electromagnetic stirring of the melt in metallurgy and for crystal growing, when rotating magnetic field homogenize of varied metal alloy and fine metal. For more details about electromagnetic stirring see [2], [3].

Whole this process is possible to simulate by commercial software Ansys Fluent (version 13). Lorentz forces in the cylindrical container are possible to determine also by analytical formula. This analytical formula was derived in my earlier work [4].

## 2. Problem formulation

This work could be interpreted as a flowing melt inside the cylindrical container. The melt inside the container is an electrically conductive and melt flowing is driven by rotating magnetic field (Fig. 1). In this article will be descripted unsteady flows caused by rotating magnetic field in cylindrical container. In Navier-Stokes equations for flow calculations the external forces are occurred. In the flow of melt driven by a rotating magnetic field these forces are the Lorentz forces. Contours of these forces could be simulated by commercial software Ansys Fluent (version 13) or determine by analytical formula.

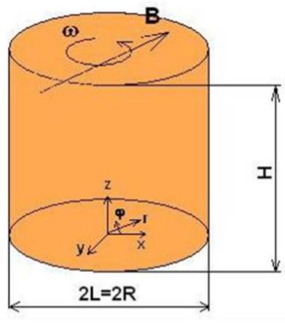


**Fig. 1:** The sketch of the flow problem

## 3. Solutions in software Ansys Fluent

MHD processes were simulated by commercial software Ansys Fluent (version 13). The Magnetohydrodynamics (MHD) module is provided as an add-on module with the standard ANSYS FLUENT licensed software. The cylindrical model and the grid were created in Ansys meshing. The grid has c. 500 000 elements (including refinement in the area of boundary layers). Boundary conditions were set to wall. The result mesh was export to Fluent, grid scale was checked (diameter of the container is 0,03 m, height of the container is 0,03 m as well) and unsteady calculation was set. Turbulence model was chosen DDES with RANS model Spalart-Allmaras. Then MHD module was unloaded and method by magnetic induction was chosen. Solution of MHD equations was added on Lorentz forces computing. Walls of the container were chosen as insulated walls, external magnetic field as AC and magnitude of the field in x and y direction $B_0$ = 0,004478 T. Magnetic induction has only components $B_x$ a $B_y$ because is assumed that vertical size of bipolar inductor is bigger than the height of the melt in the container. Because of poor and slow convergence [1] and AC electromagnetic field (and $\omega$= 439 $s^{-1}$), very small time step was chosen. In the beginning of the computing the time step was $10^{-6}$ s, at the end $5.10^{-5}$ s. Total runtime was 5 s. Because of very small time step c. 500 000 elements were chosen. Using higher number of elements of the grid is impossible because of necessity to achieve developing steady stadium.

Lorentz forces in Fluent are possible to display only in Cartesian coordinate system and only as instantaneous values, but Lorentz forces are changed because of AC field. In Fig. 2 instantaneous values of Lorentz forces are displayed. From graphs of dependence Lorentz forces on computing time approximately mean values of Lorentz forces are chosen. In Fig. 2 a) there are contours of Lorentz forces. In Fig. 2b) vectors of Lorentz forces coloured by magnitude of Lorentz forces are displayed.

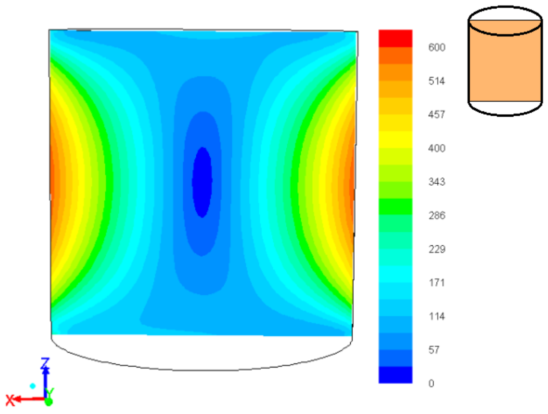


**Fig. 2a)** Contours of Lorentz forces

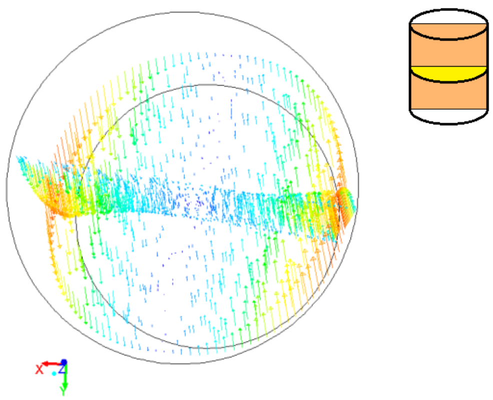

**Fig. 2b)** Vectors of Lorentz forces coloured by magnitude of Lorentz forces

Maxima of Lorentz forces are displayed by red colour, minima by violet and blue colour. With increasing distance from vertical axis values Lorentz forces are higher (in half of the container height). It is apparently caused by larger mass so that momentum is larger. Maxima of Lorentz forces are occurred on the outer walls in the half of container height. On the contrary minima of magnetic forces are occurred in the axis of container (small distance from vertical axis so that smaller momentum) and on upper and lower base (based on boundary conditions). Lorentz forces contours near upper and lower bases are affected by closeness of these bases. Even Lorentz forces part in *z* direction ($F_z$) is appeared. Lorentz forces derived from analytical formula the part of Lorentz forces in *z* direction is zero. It is caused by some simplifications in boundary conditions (scalar potential $\Phi_2 = 0$ – see [4]) and $B_z = 0$. Vectors of Lorentz forces coloured by magnitude of Lorentz forces are displayed in Fig. 2b. They are captured in some moment of computing time. The melt is driven by these forces in azimuthal direction. This direction is changed every period of AC field.

Vectors of velocity field coloured by magnitude of velocity are displayed in Fig. 4. Dominant movement (in azimuthal direction) is shown. Maxima of velocities are occurred on the outer walls of container in the half of container high. On the contrary minima of velocities are in the axis of container and on upper and lower base (based on boundary conditions). Dominant velocity is in azimuthal direction but radial and axial movement (Fig. 5) is occurred as well. These velocities are much smaller than velocities in azimuthal direction. Movement of the melt contains smaller secondary flow.

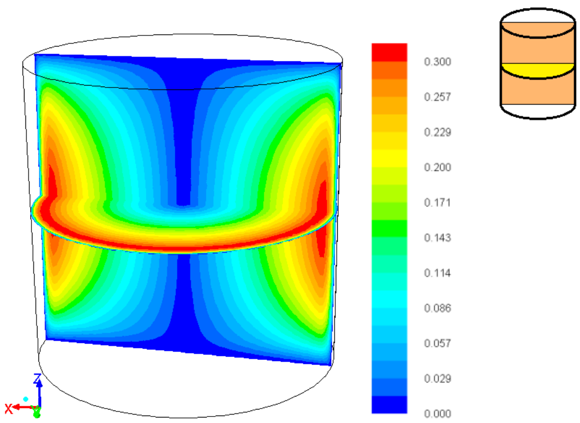


**Fig. 3** Contours of azimuthal velocity

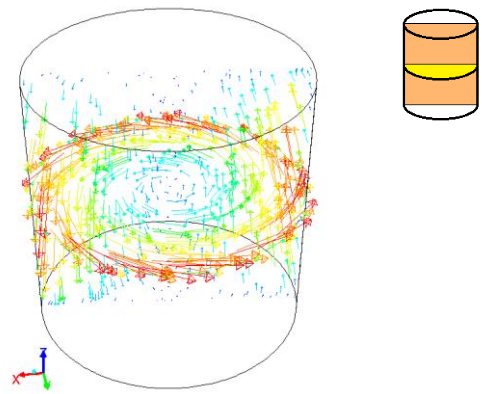

**Fig. 4** Vectors of velocity field coloured by magnitude of velocity

## 4. Solution from analytical formula

In my earlier works analytical formula of time-averaged Lorentz forces for cylindrical container were deduced [3].

$$\overline{\mathbf{f}_\varphi} = \frac{\sigma \cdot B_0^2 \cdot \varpi \cdot r}{2} \cdot \left(1 - \frac{2}{r}\sum_{i=1}^{\infty} \frac{J_1(m_i \cdot r)}{(m_i{}^2 - 1) \cdot J_1(m_i)} \cdot \frac{\sinh(m_i \cdot z) + \sinh(m_i \cdot (H - z))}{\sinh(m_i \cdot H)}\right) \qquad (1)$$

$J_1$ is Bessel function of the first kind, m are roots of equation $J_1'(m) = 0$, z is non-dimensional height of the container, r is non-dimensional radius of the container and H is non – dimensional total height of the container. Contours of Lorentz forces in Fig. 6 are similar to contours from Ansys Fluent. Differences are occurred in the area near lower and upper bases. Fluent computes forces in x, y and z directions. Deduction of forces from analytical formula has some simplifications and forces in z direction are zero.

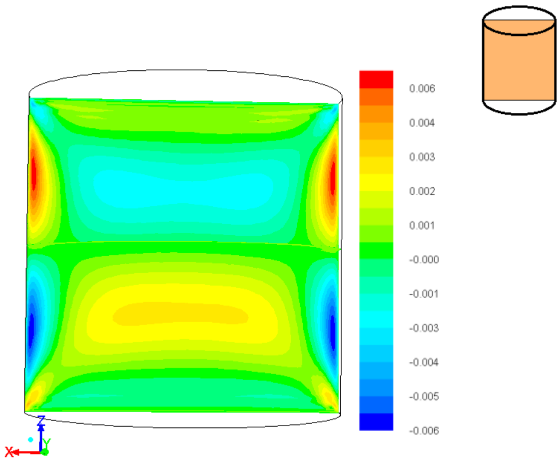


**Fig. 5** Contours of axial velocity

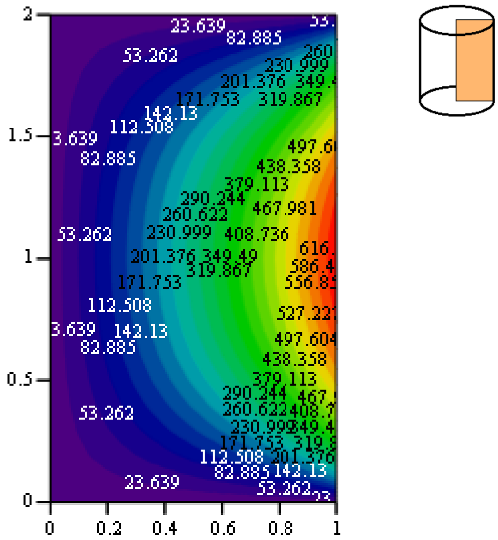


**Fig. 6** Contours of Lorentz forces from analytical formula

## Conclusion

Contours of Lorentz forces were displayed for solutions from Ansys Fluent and from analytical formula. Results of these methods were compared. Contours of Lorentz forces from Ansys Fluent are similar to contours from analytical formula. Differences are occurred in the area near lower and upper bases. Caused velocity fields in azimuthal and axial direction were displayed and described as well.

## References

[1] ANSYS FLUENT 12.0 Magnetohydrodynamics (MHD) Module Manual, April 2009

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