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NUMERICAL SOLUTION OF INVISCID AND VISCOUS FLOWS IN INTERNAL AERODYNAMICS

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This work deals with the numerical solution of 2D inviscid and viscous (laminar) compressible flows in a DCA 8% cascade achieved by the finite volume method using a multistage Runge-Kutta method with Jameson's artificial dissipation on non-orthogonal structured grids. The results are discussed and compared with other similar ones and experiment.

Keywords: transonic flow, DCA cascade, Runge-Kutta

Introduction

The goal of the work was to achieve the experience and knowledge in the field of a numerical simulation of inviscid and viscous (laminar) compressible flows and their applications in the field of turbulence modelling etc.

1. Mathematical model

Considering a 2D flow of a viscous (laminar) compressible fluid, authors have used the system of the Navier-Stokes equations

$$W_t+\mathbf{F}_x+\mathbf{G}_y=\mathbf{R}_x+\mathbf{S}_y$$

and for a simulation of an inviscid case $\boldsymbol{\eta}=0$, the system of the Euler equations

$$W_t+\mathbf{F}_x+\mathbf{G}_y=0$$

where

$$\mathbf{W}=(\boldsymbol{\rho},\boldsymbol{\rho}\mathbf{u},\boldsymbol{\rho}\mathbf{v},\mathbf{e})^T,$$
$$\mathbf{F}=(\boldsymbol{\rho}\mathbf{u},\boldsymbol{\rho}\mathbf{u}^2+\mathbf{p},\boldsymbol{\rho}\mathbf{u}\mathbf{v},(\mathbf{e}+\mathbf{p})\cdot u)^T,\mathbf{G}=(\boldsymbol{\rho}\mathbf{v},\boldsymbol{\rho}\mathbf{u}\mathbf{v},\boldsymbol{\rho}\mathbf{v}^2+\mathbf{p},(\mathbf{e}+\mathbf{p})\cdot v)^T,$$
$$\mathbf{R}=\left(\mathbf{0},\tau_{\mathbf{xx}},\boldsymbol{\tau}_{\mathbf{xy}},\mathbf{u}\boldsymbol{\tau}_{\mathbf{xx}}+\mathbf{v}\boldsymbol{\tau}_{\mathbf{xy}}+\boldsymbol{\lambda}\mathbf{T}_x\right)^T,\mathbf{S}=\left(\mathbf{0},\tau_{\mathbf{xy}},\boldsymbol{\tau}_{\mathbf{yy}},\mathbf{u}\boldsymbol{\tau}_{\mathbf{xy}}+\mathbf{v}\boldsymbol{\tau}_{\mathbf{yy}}+\boldsymbol{\lambda}\mathbf{T}_y\right)^T$$

and the systems are closed by the equation of state in the following form

$$\mathbf{p}=(\mu-1)\left[e-\frac{1}{2}\rho(u^2+\mathbf{v}^2)\right].$$

All variables were considered dimensionless.

2. Numerical method and scheme

The multistage Runge-Kutta method has been used in the form of a numerical scheme of the finite volume method on non-orthogonal structured grids of quadrilateral cells D_{ij}.

$$\mathrm{ResW}_{\mathrm{ij}}^{(r)} = \frac{1}{D_{\mathrm{ij}}}\sum\left(\tilde{F}_k \Delta \mathrm{y}_k - \tilde{G}_k \Delta \mathrm{x}_k\right)$$

$$W_{\mathrm{ij}}^{(0)} = \mathrm{W}_{\mathrm{ij}}^{n}$$

$$W_{\mathrm{ij}}^{(\mathrm{r+1})} = \mathrm{W}_{\mathrm{ij}}^{(0)} - \alpha_r \Delta \mathrm{t}\mathrm{ResW}_{\mathrm{ij}}^{(r)} + \mathrm{AD}\left(W_{\mathrm{ij}}^{n}\right), \mathrm{r}=0,1,2$$

$$W_{\mathrm{ij}}^{\mathrm{n+1}} = \mathrm{W}_{\mathrm{ij}}^{(3)}$$

$$\alpha_{0,1} = 0.5, \alpha_2 = 1$$

The scheme was extended by including Jameson's artificial dissipation to improve the stability of the method.

$$\mathrm{AD}\left(W_{\mathrm{ij}}^{n}\right) = \mathrm{C}_1\psi_1\left(W_{i-1\mathrm{j}}^{n} - 2\mathrm{W}_{\mathrm{ij}}^{n} + \mathrm{W}_{\mathrm{i+1j}}^{n}\right) + \mathrm{C}_2\psi_2\left(W_{\mathrm{ij-1}}^{n} - 2\mathrm{W}_{\mathrm{ij}}^{n} + \mathrm{W}_{\mathrm{ij+1}}^{n}\right)$$

where

$$\psi_1 = \frac{\mid p_{i-1\mathrm{j}}^{n} - 2\mathrm{p}_{\mathrm{ij}}^{n} + \mathrm{p}_{\mathrm{i+1j}}^{n} \mid}{\mid p_{i-1\mathrm{j}}^{n} \mid + \mid p_{\mathrm{ij}}^{n} \mid + \mid p_{\mathrm{i+1j}}^{n} \mid}, \psi_2 = \frac{\mid p_{\mathrm{ij-1}}^{n} - 2\mathrm{p}_{\mathrm{ij}}^{n} + \mathrm{p}_{\mathrm{ij+1}}^{n} \mid}{\mid p_{\mathrm{ij-1}}^{n} \mid + \mid p_{\mathrm{ij}}^{n} \mid + \mid p_{\mathrm{ij+1}}^{n} \mid}$$

A convergence to the steady state was followed by $\log L_2$residual defined by

$$\mathrm{RES} = \sqrt{\frac{1}{N}\sum\left(\frac{W_{\mathrm{ij}}^{\mathrm{n+1}} - W_{\mathrm{ij}}^{n}}{\Delta \mathrm{t}}\right)}$$

where $N$is a number of all cells in the computational domain.

3. Formulation of the problems

The authors took in account numerical simulations of 2D inviscid and viscous compressible flows and both have been solved in a computational domain that represents a DCA 8% cascade and its outlines are shown in Fig. 1. A left and right outline respectively is an inlet and outlet of the domain respectively. A bottom and top outlines are divided into two straight lines that mean a free wall – a part of boundary where was applied periodicity condition – and a curve that means a bottom/top part of DCA 8% profile – there were prescribed boundary conditions for a solid wall according to the mentioned type of flow (an inviscid or a viscous flow).

3.1 Boundary Conditions

- **Inlet:** $\rho_1 = 1, u_1 = \mathrm{M}_1\cos\alpha, \mathrm{v}_1 = \mathrm{M}_1\sin\alpha$, $\mathrm{p}_1$was extrapolated from the flow field and $e_1$was calculated using the equation of state, where α is angle of attack.

- **Outlet:** p_2 was prescribed, $\rho_{2,} u_{2,} v_2$ were extrapolated from the flow field and e_2 was calculated using the equation of state, Boundary conditions on the solid wall and periodicity conditions were implemented by using virtual cells adjoined from outside of computational domain and values of the variables inside of them were prescribed to obtain the desired effect.

- **Solid wall:** velocity components were prescribed so that a sum of velocity vectors equals to zero u=v=0(a viscous flow) or equals to zero in their tangential component(u,v)$\vec{n} = 0$(an inviscid flow).
- **Periodicity:** a value of the variable in a cell at the bottom part of boundary corresponds to a value in a cell from the flow field near the top part boundary.

Initial conditions were prescribed to comply with the inlet conditions.

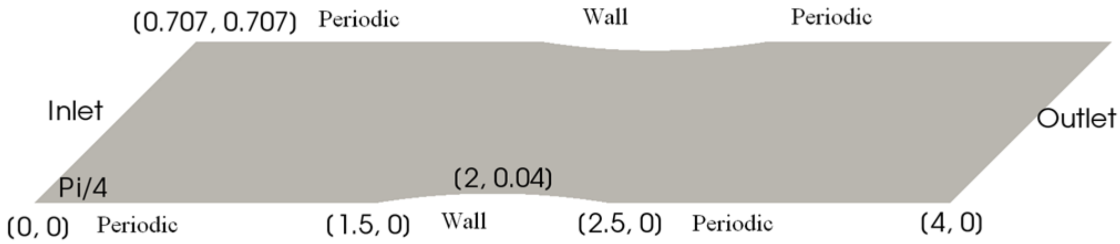

Fig. 1: Computational domain with boundary conditions applied (source: J. Trefilík)

4. Numerical results

The authors used two non-orthogonal structured grids with quadrilateral cells – 130 cells in the direction of the axis x and 50 cells (inviscid flows) and 120 cells (laminar flows) in the direction of the axis y (of the a straight line connecting inceptions of a low and an upper DCA 8% profile). Of course, in the case of viscous flows authors made such a convenient refinement of mesh near the solid walls for a better detection of viscosity influence.

The authors took in account several values of inlet Mach and Reynolds numbers, and angles of attack to obtain results comparable with the experimental data and other similar numerical solution that would verify and satisfy a use of a software develevoped by the authors and used the multistage Runge-Kutta method.

In the **Figs. 2a** and **4a** we can see some results for 2D transonic flows of inviscid compressible fluid that have been compared to results by P. Pořízková – Figs. 2b and **4b** – and In the **Figs. 5a** and **5b** results for 2D viscous flows. A good agreement was found both with the results by P. Pořízková and with the experimets.

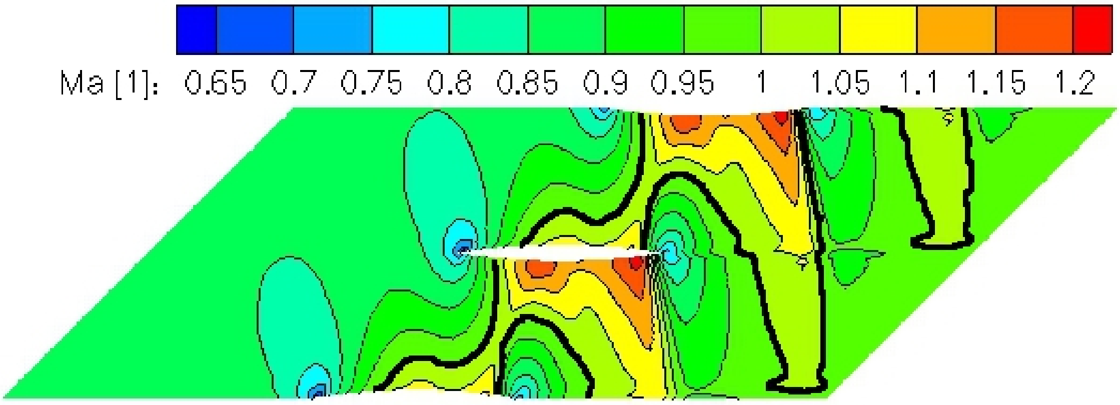

Fig. 2a: An inviscid compressible flow at $M_1 = 0.92$,α=2°- multistage Runge-Kutta method, mesh: 130x50 cells

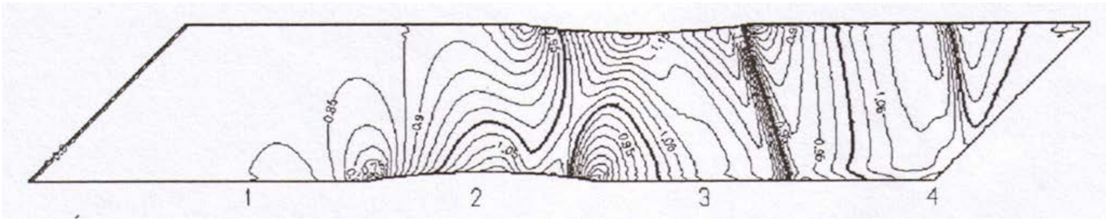

Fig. 2b: An inviscid compressible flow at $M_1 = 0.92$,α=1.2°- MacCormack scheme, mesh: 150x50 cells, author: P. Pořízková [3]

Fig. 3: A compressible flow at $M_1 = 0.92$,α=1.2- experiment of IT AS CR, source: [3]

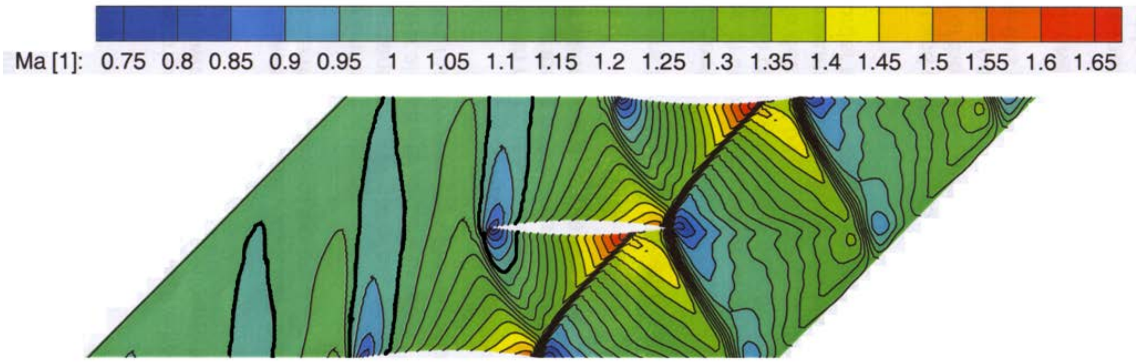

Fig. 4a: An inviscid compressible flow at $M_1 = 1.12$,α=0.5°- multistage Runge-Kutta method, mesh: 130x50 cells

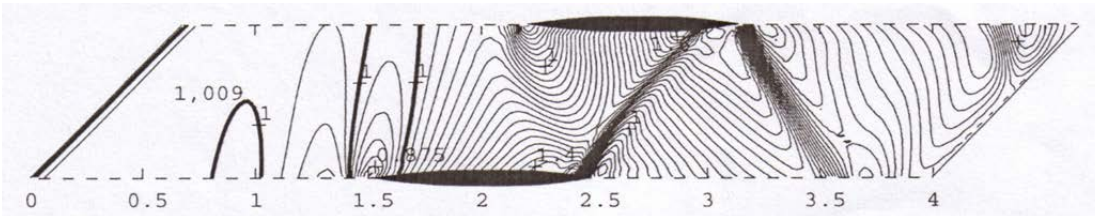

Fig. 4b: An inviscid compressible flow at $M_1 = 1.12$,α=0°- composite scheme, mesh:150x30 cells, author: P. Pořízková [3]

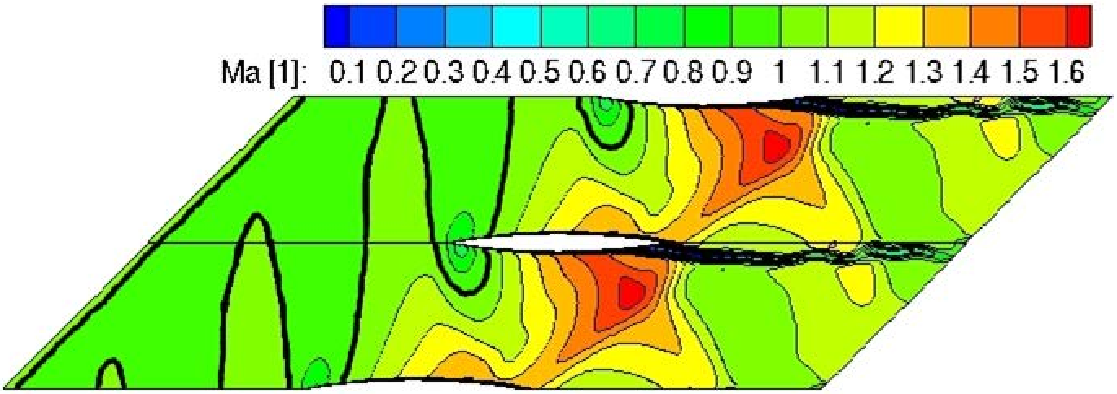

Fig. 5a: A laminar compressible flow at $M_1 = 1.1$,Re=$2.1 \cdot 10^6$,α=0°- multistage Runge-Kutta method, mesh: 170x120 cells

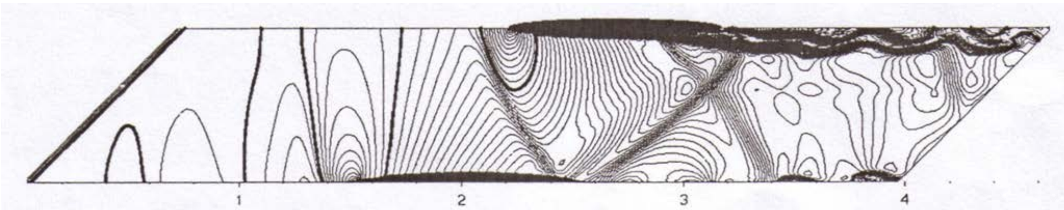

Fig. 5b: A laminar compressible flow at $M_1 = 1.1$,Re=$2.1 \cdot 10^6$,α=0°- MacCormack scheme, mesh: 150x30 cells, author: P. Pořízková [3]

Conclusions

This article presents some results achieved by using own software with the implemented FVM multistage Runge-Kutta method and added Jameson's artificial dissipation for a simulation of a 2D transonic flow of an inviscid and laminar compressible fluid in the DCA 8% cascade. Numerical results shows a good agreement with other numerical results (e.g. P. Pořízková [3]) and experimental results carried out at the Institute of Thermodynamics AS CZ.

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