

30. Setkání kateder mechaniky tekutin a termomechaniky



22.-24.6. 2011

Špindlerův Mlýn

Jednotlivý příspěvek ze sborníku



TECHNICKÁ UNIVERZITA V LIBERCI



evropský
sociální
fond v ČR



EVROPSKÁ UNIE



MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



OP Vzdělávání
pro konkurenceschopnost

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Criteria of liquid wall film atomization

Stanislav KNOTEK¹, Miroslav JÍCHA²

- ¹ Ing. Stanislav Knotek, FSI, VUT v Brně, Technická 2896/2, 616 69 Brno, knotek@fme.vutbr.cz
² Prof. Ing. Miroslav Jícha, CSc., FSI, VUT v Brně, Technická 2896/2, 616 69 Brno, jicha@fme.vutbr.cz

Abstrakt: Criteria derived in case of thin liquid wall film atomization are presented. The criteria are distinguished according to a leading variable in the criteria depending on the film thickness and the criteria whose control variable is a wavelength of waves on the film surface.

1. Introduction

The issue of liquid wall film atomization have been studied mostly in the context of the annular type of flow in pipeline. The subject of this paper is to present basic approaches to the description of atomization conditions and to summarize selected criteria derived in case of flat wall films. This option allows using of standard procedures used in the prediction of hydrodynamic instability preceding the atomization and is also better adapted to applications, where wall curvature with respect to low film thickness can be neglected. Nevertheless criteria based primarily on the piping systems are also given for comparison.

For clarity, the criteria are distinguished into two types. The criteria of the first group depend on the wavelength of the waves from the top of which the fragments of fluid are atomized. The criteria of the second group take into account the thickness of the liquid film. In the first case, the basic principle of the theory of Kelvin-Helmholtz instability is used and the latter criteria are derived from the definition of Weber number.

2. Criteria depending on wavelength

The fundamental basis for the derivation of criteria of the liquid surface atomization is the theory of Kelvin-Helmholtz instability. The power balance of stabilizing and destabilizing forces leads to a condition of neutral stability

$$\hat{P}_{SR} + g\rho_L + \sigma k^2 = 0, \quad (1)$$

where ρ_L and σ is the density and surface tension respectively, g is the gravitational acceleration and $k=2\pi/\lambda$ is the wavenumber of wave with wavelength λ . Quantity \hat{P}_{SR} is defined via the pressure fluctuation on the wave surface

$$P'_S = ae^{kC_R t} [\hat{P}_{SR} \cos k(x - C_R t) - \hat{P}_{SI} \sin k(x - C_R t)], \quad (2)$$

where a is the amplitude of the wave described by a relation for the interface displacement

$$h' = a \cos k(x - C_R t) \quad (3)$$

from the equilibrium $\bar{h} = h - h'$ depending on the time-space coordinates (x,t) and phase velocity C_R , see Hanratty (1983).

From equation (2) and the physical nature of the problem is obvious, that the quantity \hat{P}_{SR} has the meaning of the amplitude of pressure fluctuation over the wavy surface and its value is dependent on the geometric configuration, i.e. the ratio of wavelength to wave amplitude λ/a and thickness of the considered channel B , and also on the air velocity.

Theory of the Kelvin-Helmholtz instability assumes a uniform velocity profile of the gas flow, which by Hanratty (1983) leads to a formula

$$\hat{P}_{SR}^{KH} = -(U_G - C_R)^2 k \rho_G, \quad (4)$$

where U_G and ρ_G is the average air velocity and gas density respectively. Form the configuration of the considered phenomena, it is clear that the assumption of uniform profile is not entirely adequate and therefore it can be assumed that real values of \hat{P}_{SR} will differ from the course defined by the equation (4).

Authors in [9] give the relation for \hat{P}_{SR} assuming real velocity profile and solving Orr-Sommerfeld equation, for details see [4]. The resulting relationship has the form

$$\hat{P}_{SR} = 0.131 \rho_G U_G^2 k \left(\frac{kB}{2} \right)^{-0.627} Re_G^{0.229} \quad (5)$$

where $Re_G = BU_G/\nu_G$ is Reynolds number of air flow and B is height of air space above the liquid surface.

In the following we give a criteria derived from the first condition (1) by substituting for \hat{P}_{SR} from relations (4) and (5).

By substituting (4) into (1) and assuming $U_L \approx C_R$ the neutral stability condition is obtained

$$U_G - U_L = \left(\frac{\sigma k^2 + g \rho_G}{k \rho_G} \right)^{1/2}. \quad (6)$$

Note that the derivation according to the classical theory of Kelvin-Helmholtz instability, see [6], leads to the condition

$$\begin{aligned} U_G - U_L \\ = \left(\frac{[\sigma k^2 + (\rho_L - \rho_G)g](\rho_L - \rho_G)}{k \rho_L \rho_G} \right)^{1/2}. \end{aligned} \quad (7)$$

Evidently, the criterion (6) follows from (7) for $\rho_L \gg \rho_G$.

By substituting (5) into (1) the criterion (8) is obtained.

$$U_G = \left[\frac{(\sigma k^2 + \rho_L g) \left(\frac{B}{\nu_G} \right)^{0.229}}{0.131 \rho_L \left(k \frac{B}{2} \right)^{-0.627}} \right]^{\frac{1}{2.229}} \quad (8)$$

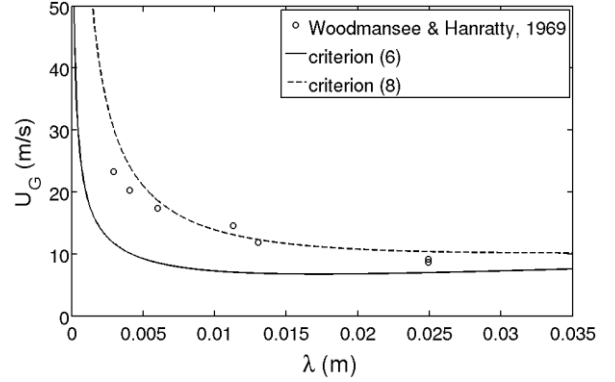


Figure 1: Comparison of criteria (6) and (8) with experimental data [9].

As is shown in the figure 1, classical criterion of the Kelvin-Helmholtz instability (6) significantly underestimates the critical velocities. This directly corresponds with the pressure amplitude formula (4) used to derive the criterion (8) which is not particularly good approximation for short wavelengths.

At the end of the paragraph note that so far mentioned criteria define the critical velocity in dependence on the wavelength of the expected waves. However, this parameter is not a priori known in the real case and it can be expected, see [1], that the wavelength depends on the liquid film thickness. Authors in the article [9], on the basis of experiments, assume a wavelength $\lambda = 5h_p$, where h_p is the height of a solitary wave from the top of which atomization occurs. So in this case, the problem of critical velocity is dependent on the prediction of parameter h_p , but this is not a trivial task and the article does not give any solution. Therefore, these criteria represent only a partial solution of the atomization problem and moreover do not reflect the influence of the film thickness. Therefore, further we shall examine the criteria without these deficiencies.

3. Criteria depending on film thickness

The basic parameter which can quantify the affinity for the atomization depending on the film thickness is the Weber number

$$We = \frac{\rho U^2 h}{\sigma} \quad (9)$$

where ρ and U is the density and the film surface velocity respectively, see [6], or the gas density and relative velocity of the gas to the film surface respectively, see [9].

In the latter case, the criterion of atomization can be easily derived in the form

$$U_G = C_R + \sqrt{\frac{We_{cr} \sigma}{\rho_G h}}. \quad (10)$$

The problem of quantification of the critical velocity is now moved to determination of the critical Weber number We_{cr} . Authors in [9], according to experiments, distinguish the critical Weber number calculated for the basic film thickness, $We_{cr} \approx 1.5$, and for the height of a solitary wave, $We_{cr} \approx 5.5$.

For Weber numbers defined by the surface velocity of the film, see [6], the authors derive the condition of atomization on the basis of equality of shear stress on the film surface

$$\tau_i = f_i \frac{\rho_G}{2} U_G^2 = \mu_L \frac{U_i}{h}, \quad (11)$$

where f_i and U_i is the shear stress coefficient and liquid surface velocity respectively. By eliminating U_i using (10), the criterion can be derived from (8)

$$U_G = \left(\frac{2\sqrt{We_{cr}} \mu_L}{f_i h \rho_G} \sqrt{\frac{\sigma}{h \rho_L}} \right)^{\frac{1}{2}}, \quad (12)$$

where the authors consider the value $We_{cr}=3$, which identified Miles, see [7], from conditions for hydrodynamic stability. The values of critical velocity defined by equation (12) are

now dependent on the determination of shear stress coefficient. The authors used the relationship for the annular flows in pipelines with hydraulic diameter D_h :

$$f_i = 0.005 \left(1 + 300 \frac{h}{D_h} \right). \quad (13)$$

In case of film over the flat wall the dependence

$$f_i = 0.0002 Re_L + 0.01 \quad (14)$$

can be used according to [8]. $Re_L = hu_a/v_L$ is Reynolds number of liquid flow. By substituting the formula for liquid surface velocity

$$U_i = \left(\frac{U_G}{8.74} \right)^{\frac{7}{4}} \left(\frac{2v_G}{B} \right)^{\frac{1}{4}} \frac{h \rho_G}{\mu_L} \quad (15)$$

derived in [5] into (11) and by approximation $u_a = U_i/2$ in (14) the criterion

$$U_G = \left[\frac{2\rho_L^2 v_L^3}{h^3 \rho_G^2} \frac{\sqrt{\frac{\sigma We_{cr}}{h \rho_L}}}{10^{-5} \left(\frac{1}{8.74} \right)^{\frac{7}{4}} \left(\frac{2v_G}{B} \right)^{\frac{1}{4}}} \right]^{\frac{4}{15}} \quad (16)$$

can be derived based on (9).

Another criterion can be derived directly from the definition of Weber number (9) by putting the film surface velocity from equation (15):

$$U_G = 8.74 \left(\frac{We_{cr} \sigma \mu_L}{h^2 \rho_L \rho_G} \right)^{\frac{4}{7}} \left(\frac{B}{2v_G} \right)^{\frac{1}{7}}. \quad (17)$$

In Figure 6 are plotted the critical velocities of the gas depending on the liquid film thickness according to the criteria (10), (12) for shear stress coefficient defined by (13), (16) and (17) for the height of the channel, eventually the hydraulic diameter, $B = 0.025$ m. The comparison of these criteria results in relatively good agreement between the two discussed approaches especially for low film thickness. The derived criteria (16) and (17) give not so good prediction in comparison with criterion (12) although the shear stress coefficient (14)

should be more appropriate than (13) in the case of flat wall.

As was discussed in the preceding section other criterion may be obtained from some criterion depending on wavelength by choosing the ratio between the film thickness and wavelength. According to experimental data, see [9], $h_p \approx 3-5 h_0$, where h_0 is the basic film thickness. Then for $\lambda = 5h_p$, according to [9], we obtain an approximate value $\lambda/h_0 \approx 15-25$. Thus, the last criterion is constructed from (8) for $\lambda/h_0 = 20$ and depicted in figure 2 numbered as criterion (18). It seems that this give the best fit of experimental data among all discussed criteria especially for longer wavelength i.e. greater film thickness.

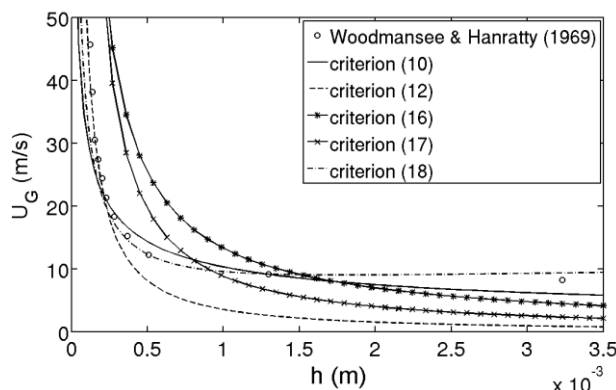


Figure 2: Comparison of criteria (10), (12), (13), (16), (17) and (18) with experimental data [9].

4. Conclusion

The aim of the article was the presentation of two basic approaches to the design of atomization criteria of liquid film forming on the wall of negligible curvature. These criteria are derived from the power balance based on the theory of Kelvin-Helmholtz instability and on the definition of a critical Weber number. In the former case, the criteria are dependent on the wavelength of the waves occurring on the film surface. This attitude requires the addition of empirical or computational models of the expected wavelengths, depending on the thickness of the liquid film thickness and air stream velocity. Criteria for the latter type do not require information about wavelength and

also reflect the effect of the film thickness. The problem of the adequacy of these criteria depends on determining the critical Weber number or shear stress or velocity of liquid on the film surface. The comparison of different criteria results in relatively good agreement between the two discussed approaches and also between the criteria derived from the definition of Weber's relationship with the additional models mentioned above. More detailed evaluation of presented criteria is limited by the insufficient number of experimental data, which motivates the basis for further study of the discussed issue.

Acknowledgement:

The article was supported by the project GAČR GA101/08/0096 and by FSI-S-11-6.

5. References

- [1] ASALI, K.C., HANRATTY, T. J.: Ripples generated on liquid film at high gas velocities, *Int. Multiphase Flow*, 19, pp. 229-243, 1993.
- [2] BONDI, H. On the generation of waves on shallow water by wind, *Proc. R. Soc. Lond. A* 181, 67-71, 1942.
- [3] HANRATTY, T. J. Interfacial instabilities caused by air flow over a thin liquid layer. *Waves on Fluid Interfaces* (Edited by Meyer, R.E.), New York, Academic Press, pp. 221-259, 1983.
- [4] COOK, G. W. Shear stress and pressure variation over small amplitude waves, M.S. thesis in Chemical Engineering, University of Illinois, 1967.
- [5] CRAIK, A.D.D.: Wind-generated waves in thin liquid films, *J. Fluid Mechanics*, 26, pp. 369-392, 1966.
- [6] KIM, B.H.; PETERSON, G.P. Theoretical and physical interpretation of entrainment phenomenon in capillary-driven heat pipes using hydrodynamics instability theories, *Int. J. Heat Mass Transfer*, 37, 17, pp. 2647-2660, 1994.
- [7] MILES, J. W. The hydrodynamic stability of a thin film of liquid, *J. Fluid Mech.* 8, pp. 593-610, 1960.
- [8] MYIA, M., WOODMANSEE, D.E., HANRATTY, T.J.: A model for roll waves in gas-liquid flow, *Chem. eng. Science* 21, pp. 1915, 1971.
- [9] WOODMANSEE, D.E.; HANRATTY, T.J. Mechanism for the removal of droplets from a liquid surface by a parallel air flow, *Chemical Engineering Science*, 24, pp. 299-307, 1969.