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Numerical solution of 2D turbulent flows

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The work aims at developement of numerical methods for simulation of transonic turbulent flows in various configurations namely through the DCA 8% cascade and over a two-dimensional 18% thick circular-arc biconvex airfoil. Results of various numerical experiments modelling the viscous and inviscid flows are presented. For turbulence modelling a zero equation algebraic Baldwin-Lomax model along with two equations standard k-ω and modified (TNT) models were employed and the results of these calculations are compared.

Key words: turbulence modelling, finite volume method, k-ω model

Introduction

In this work we aim to model subsonic and transonic turbulent flows in internal aerodynamics. The numerical solution is carried out by using a finite volume method based on MacCormack scheme. The turbulence phenomena is described by Reynolds averaged Navier-Stokes equations which are closed by 3 turbulence models.

Mathematical models

The two dimensional flow of a viscous compressible fluid is described by the systém of Navier Stokes equations.

$$W_t + F_x + G_y = R_x + S_y \tag{1}$$

where

$$\begin{aligned} W &= (\rho, \rho u, \rho, e)^T \\ F &= (\rho u, \rho u^2 + p, \rho uv, (e+p)u)^T \\ G &= (\rho v, \rho uv, \rho v^2 + p, (e+p)v)^T \end{aligned} \tag{2}$$

and

$$\begin{aligned} R &= (0, \tau_{xx}, \tau_{xy}, u\tau_{xx} + v\tau_{xy} + kT_x)^T \\ S &= (0, \tau_{xy}, \tau_{yy}, u\tau_{xy} + v\tau_{yy} + kT_y)^T \end{aligned} \tag{3}$$

with shear stresses given for the laminar flow by equations

$$\tau_{xx} = \frac{2}{3}\eta\left(2u_x - v_y\right) \quad \tau_{xy} = \eta\left(u_y + v_x\right) \quad \tau_{yy} = \frac{2}{3}\eta\left(-u_x + 2v_y\right) \tag{4}$$

This system is closed by the equation of state

$$p = (\kappa - 1)\left[e - \frac{1}{2}\rho\left(u^2 + v^2\right)\right] \tag{5}$$

In the above given equations, ρ denotes density, *p* is pressure, *T* is temperature, η is dynamical viscosity, *k* is thermal conductivity coefficient, *e* is total energy per unit volume and *u*, *v* are components of velocity in the direction of axis *x*, *y*. The parameter κ = 1.4 is the adiabatic exponent.

By assuming that η=0, we obtain the model of inviscid compressible flow which is represented by a system of so called Euler equations:

$$W_t + F_x + G_y = 0 \tag{6}$$

For modelling of turbulent flow, the system of the RANS (Reynolds Averaged Navier-Stokes) equations closed by a turbulence model is used. The system of averaged Navier-Stokes equations is formally the same as above, but this time the flow parameters represent only mean values in the Favre sense, see Favre (1965). The shear stresses are given for the turbulent flows by equations

$$\tau_{xx} = \frac{2}{3}(\eta + \eta_t)(2u_x - v_y) \quad \tau_{xy} = (\eta + \eta_t)(u_y + v_x) \quad \tau_{yy} = \frac{2}{3}(\eta + \eta_t)(-u_x + 2v_y) \tag{7}$$

where η_t denotes the turbulent dynamic viscosity according to the Boussinesq hypothesis.

The Reynolds number is defined by $Re = u_\infty L / \eta_\infty$ and the Mach number by M = $(q/a)^{1/2}$ where q = $(u^2 + v^2)^{1/2}$ and a is the local speed of sound. All computations were realized using dimensionless variables with reference variables given by inflow values. The reference length L is given by the width of the computational domain.

Turbulence models

Baldwin-Lomax model

Algebraic models are based on the model proposed for the boundary-layer flows by Cebeci and Smith. Baldwin-Lomax model is its modification applicable for general turbulent shear flows. The boundary layer is divided into two regions. In the inner (nearest to the wall) part, the turbulent viscosity is given by

$$\eta_{ti} = \rho F_D^2 \kappa^2 y^2 |\Omega| \tag{8}$$

where Ω is the vorticity, which is in the 2D flow determined by

$$\Omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \tag{9}$$

The turbulent viscosity in the outer region is given by

$$\eta_{to} = \alpha\rho C_{cp} F_w F_k \tag{10}$$

where C_{cp} is a constant. Function F_w is determined by the relation

$$F_w = y_{max} F_{max} \tag{11}$$

for F_w being the maximum of the function

$$F = yF_D \left|\Omega\right| \tag{12}$$

and y_{max} the distance from the wall in which $F(y_{max}) = F_{max}$ holds and

$$F_k = \left[1 + 5.5\left(C_{KL}\frac{y}{y_{max}}\right)^6\right]^{-1} \tag{13}$$

The Baldwin-Lomax model (1978) contains following values of the constants: $\kappa = 0.4$; $A^+ = 26$; $\alpha = 0.0168$; $C_{cp} = 1.6$; $C_{KL} = 0.3$.

k-ω model

Two-equation models are based on transport equations for two characteristic scales of turbulent motion, mostly for the turbulent energy *k* and dissipation rate ε, often used in the form of specific dissipation rate $\omega = \varepsilon/k$. These characteristics are computed from transport equations. Turbulent viscosity is defined as

$$\mu_t = \gamma^* \rho \frac{k}{\omega} \tag{14}$$

The **standard Wilcox k-omega model** is defined by the equations

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial\left(\rho u_j k\right)}{\partial x_j} = P_k + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma^* \mu_t\right)\frac{\partial k}{\partial x_j}\right] - \beta^* \rho k \omega \tag{15}$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial\left(\rho u_j \omega\right)}{\partial x_j} = \gamma \frac{\omega}{k} P_k + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma \mu_t\right)\frac{\partial \omega}{\partial x_j}\right] - \beta \rho \omega^2 \tag{16}$$

where $P_k = \tau_{ij}\partial u_i/\partial x_j$ represents the production of turbulent energy. Model coefficients are given by values: $\gamma^* = 1$, $\gamma =$ 5/9, β = 3/40, β^* = 9/100, σ = ½ and σ^* = ½.

Kok [8] suggested following modification of the standard form denoted as **TNT variant of k-ω model.**

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = P_k + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \mu_t\right)\frac{\partial k}{\partial x_j}\right] - \beta^* \rho k \omega \tag{17}$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \alpha \frac{\omega}{k} P_k + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_\omega \mu_t\right)\frac{\partial \omega}{\partial x_j}\right] - \beta \rho \omega^2 + C_D \tag{18}$$

where

$$C_D = \sigma_d \frac{\rho}{\omega} \max\left\{\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, 0\right\}, \quad \alpha_\omega = \frac{\beta}{\beta^*} - \frac{\sigma_\omega \kappa^2}{\sqrt{\beta^*}}$$

and $\kappa = 0.41$, $\beta = 3/40$, $\beta^* = 9/100$, $\sigma_\omega = 0.5$, $\sigma_k = 0.666$.

Numerical method

The finite volume method was used on a structured grid of quadrilateral cells D_{ij}. The Mac Cormack scheme in the cell centered form was applied in solving the system of RANS equations. The Jameson's artificial dissipation was added to increase the numerical stability.

Formulation of the problem

The outline of the computational domains is shown in figures 1 and 2:

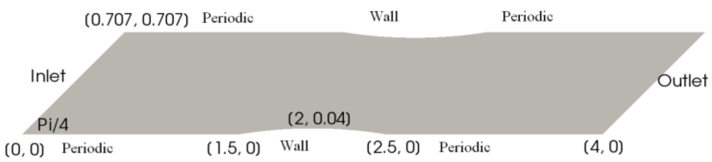

Fig. 1: test case DCA 8%

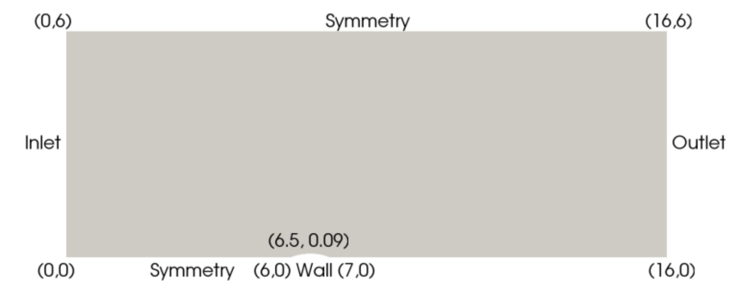

Fig 2: test case AIRFOIL

In all the test cases we imposed (normalised) Dirichlet boundary condition on the **inlet**:

$u_\infty = M_1 \cos(\alpha)$, $v_\infty = M_1 \sin(\alpha)$ and ρ_∞, e_∞ and p_∞ such that they comply with u_∞ and v_∞.

The angle α was in general a nonzero value.

On the **outlet** we prescribed only pressure $p = p_\infty$ and the rest is extrapolated from the flow field. Further on there are three other types of boundary conditions: wall, symmetry axis and periodic boundary. These conditions are implemented by using virtual cells. Such cells adjoin from outside on the boundary cells and we prescribe values of unknowns inside of them to obtain the desired effect.

Wall – viscous flow: velocity components prescribed so that the sum of velocity vectors equals zero, the rest of unknows is the same in both the virtual and the boundary cell.

Wall – inviscid flow: velocity components prescribed so that the sum of velocity vectors equals to zero in its tangential component, the rest of unknowns is the same.

Symmetry axis: same as the wall condition for inviscid flow.

Periodic boundary: taking two corresponding segments of boundary we prescribe into virtual cells of the first segment the values of unknowns contained in the boundary cells of the second and vice-versa.

Initial conditions are prescribed so that they comply with the inlet conditions.

Numerical results

First two pictures show the flow around the airfoil. Then two different setups of a flow in the DCA cascade are examined and compared with an experiment.

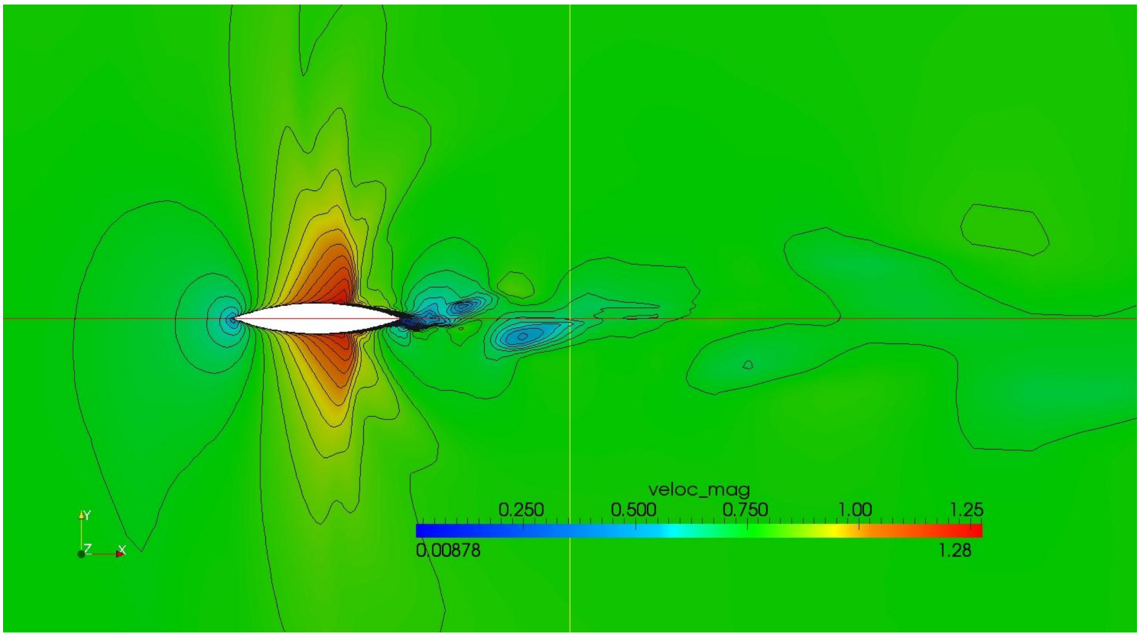

Fig 3: Compressible turbulent flow for Re = 10^7 and M_{inf} = 0.775 (basic variant of k-omega model)

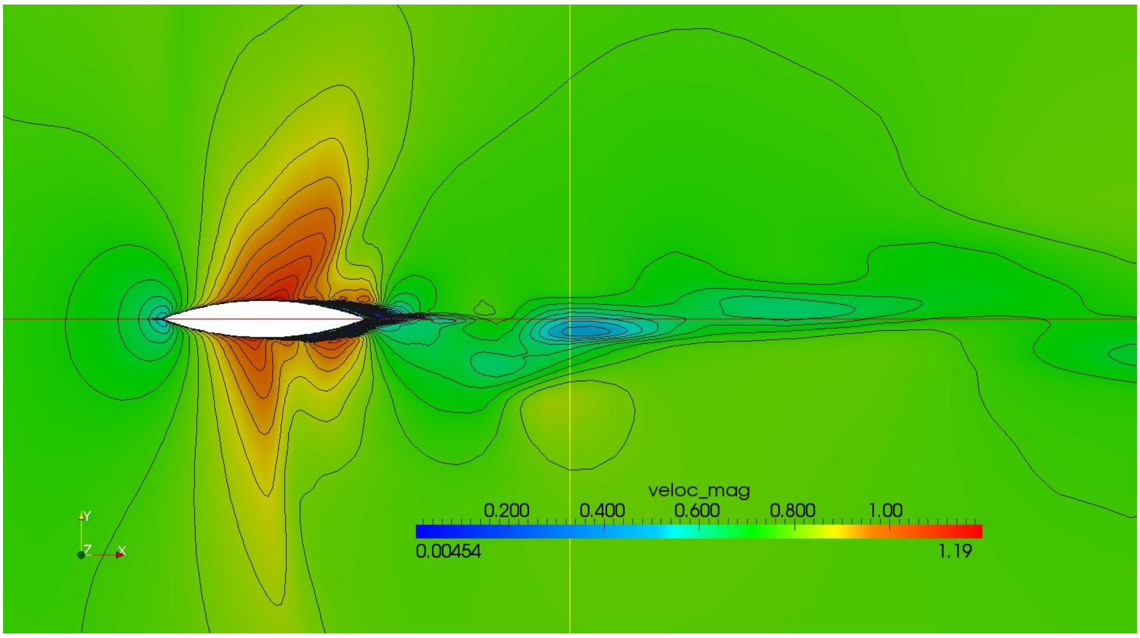

Fig 4: Compressible turbulent flow for Re = 10^7 and M_{inf} = 0.775 (Baldwin-Lomax model)

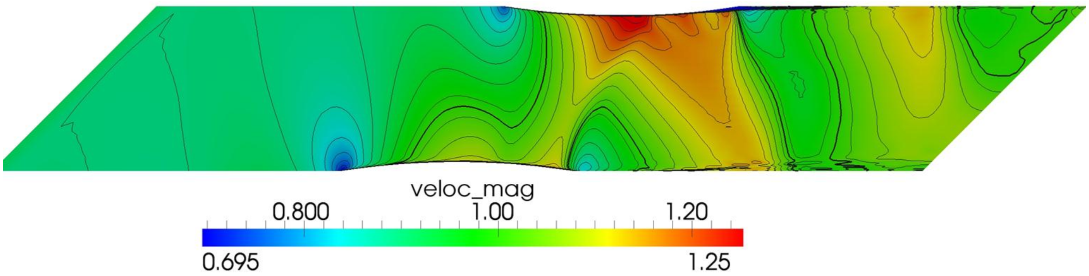

Fig 5: DCA cascade, viscous flow, M_∞=0.98, α=3.0°, Re = 10^7, Baldwin-Lomax model, Mach number isolines

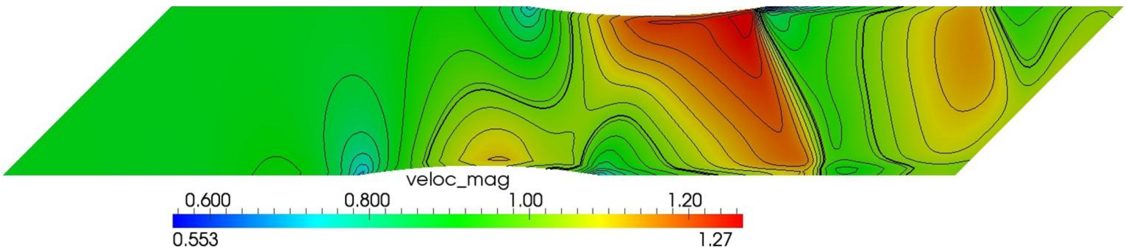

Fig 6: DCA cascade, viscous flow, M_∞=0.98, α=2.8°, Re = 10^7, k-ω model (TNT variant), Mach number isolines

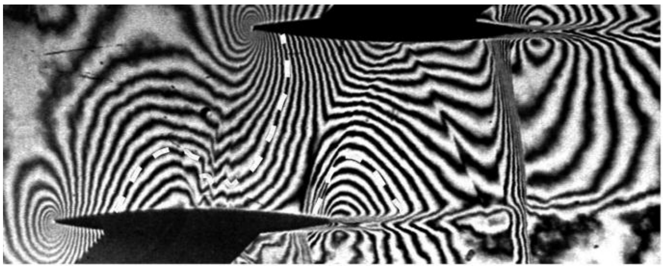

Fig 7: Experimental data, $M_{inf} = 0.863$, $\alpha=0^{\circ}$

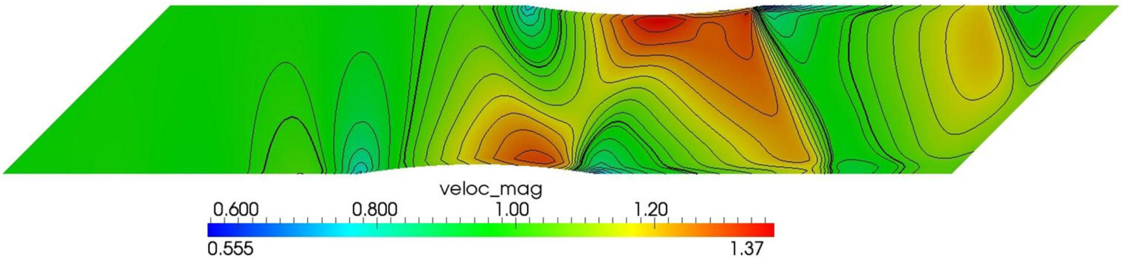

Fig 8: DCA cascade, viscous flow, $M_{\infty}=1.07$, $\alpha=2.3^{\circ}$, Re = 10^7, k-ω model (Kok variant), Mach number isolines

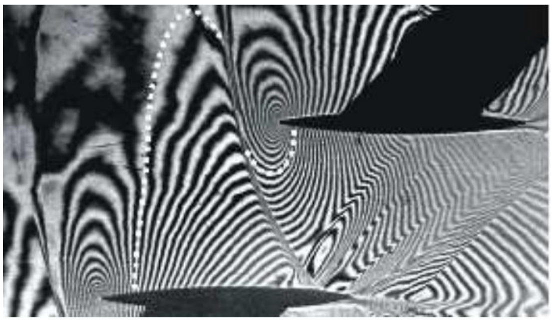

Fig 9: Experimental data, $M_{inf} = 0.982$, $\alpha=0^{\circ}$

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