30. Setkání kateder mechaniky tekutin a termomechaniky

22.-24.6. 2011

Špindlerův Mlýn

Jednotlivý příspěvek ze sborníku

TECHNICKÁ UNIVERZITA V LIBERCI

evropský sociální fond v ČR
EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Design of pump spiral based on the differential geometry

Zdeněk Sloupenský[1], František Pochylý[2]

[1] Ing. Zdeněk Sloupenský, BUT FME, Victor Kaplan Department of Fluid Engineering, Technická 2, 616 69 Brno, ysloup00@stud.fme.vutbr.cz
[2] Prof. Ing. František Pochylý, CSc., BUT FME, Victor Kaplan Department of Fluid Engineering, Technická 2, 616 69 Brno, pochyly@fme.vutbr.cz

Abstract: *A new method of designing of spiral meridional section and spiral spatial shape is introduced in this contribution. The method originates from differential geometry principles and the solution is performed in the curvilinear orthogonal net. The spiral spatial shape is based on the meridional velocity field. The aim of presented solution was achievement of proper hydraulic characteristics. This is ensured due to fluent change of the instantaneous speed of the particle. The fluency is especially endangered in the passage between the spiral and the volute throat. The spiral outlet meridional section shape smoothly changes into circular pipeline section shape in the outlet of volute throat.*

1. Introduction

The spiral design presented below is part of centrifugal pump design based on the differential geometry principles in the curvilinear coordinate system. The efficiency of the complex depends on high-quality design of each of its parts. There is also requested interconnection and reciprocity between components of designed pump. That is based on the assumptions of flow in early stages of design. The results are expected to be good foundation for further pump design performed in some CFD program or for rapid prototyping.

2. Design philosophy

The spiral meridional section does not have a universal shape. Nearly every spiral belonging to a specific type of pump has its own unique meridional section. This variability in applications is kept with defining of meridional section as Bezier surface. Equations (1) and (2) determine the Bezier surface.

$$\mathbf{r}(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n}\mathbf{r}_{ij}\binom{m}{i}u^{i}(1-u)^{m-i}\binom{n}{j}v^{j}(1-v)^{n-j} \tag{1}$$

$$\mathbf{u}=\langle 0,1\rangle \qquad \mathbf{v}=\langle 0,1\rangle \tag{2}$$

The dimensionless vectors **u**, **v** determine roughness of Bezier surface.

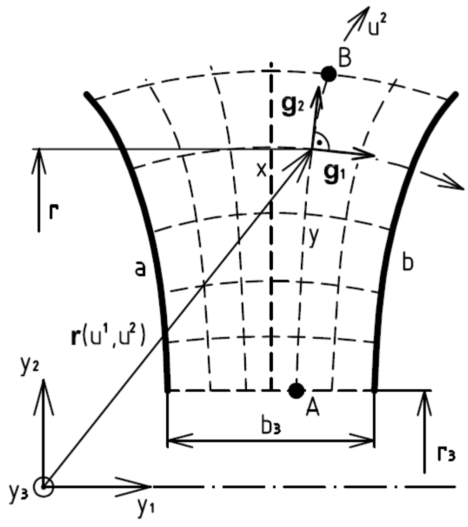

Fig. 1 Meridional section of spiral, [1]

Consider curvilinear coordinate system $u,v\in\langle 0,1\rangle; w=\varphi\in\langle 0,2\pi\rangle$ [2]. $\mathbf{g}_i$ denotes the vector tangent to appropriate coordinate curve, see Fig. 1. The calculations within the mathematical model require mutually orthogonal vectors $\mathbf{g}_i$. Nevertheless coordinate curves are not generally orthogonal.

Such orthogonalization can be performed by following equation, [3]:

$$\mathbf{c}(t) = \left(\mathbf{g}_1 - \frac{\mathbf{g}_1 \cdot \mathbf{g}_2}{\mathbf{g}_2 \cdot \mathbf{g}_2} \mathbf{g}_2 \right) \frac{du}{dt} \qquad (3)$$

Initial assumptions and boundary conditions, [3]:

$$\mathbf{c} \cdot \mathbf{g}_2 = 0 \qquad (4)$$

$$v = v_0 ; u = 0 \qquad (5)$$

3. Meridional section

There were considered three assumptions of flow for designing of impeller meridional section in [3]. These were the potential flow, the rotational flow (Francis method) and the quasi-potential flow assumptions. Only one of them is applicable for spiral. It is essential for centrifugal pump impeller design to achieve uniform distribution of meridional velocity at the outlet for any flow assumption. Thus, there is always uniform distribution of meridional velocity at the spiral inlet. Such premise gives priority to application of Francis method and also ensures interconnection between design of impeller and spiral. The design of meridional section as Bezier surface to meet requirements of rotational flow assumption is presented in [3]:

$$\frac{\mathbf{g}_1(u^1,1,u^3)}{\mathbf{g}_1(u^1,u^2,u^3)} \frac{r_2 \mathbf{g}_2(u^1,u^2,u^3)}{r(u^1,u^2,u^3)} v_{m2} = F(u^2) \qquad (6)$$

$$F(u^2) = \frac{Q}{2\pi \int_0^1 r \frac{\mathbf{g}_1}{\mathbf{g}_2} du^1} \qquad (7)$$

Equation (6) determines the system of coordinate curves for potential flow. Function $F(u^2)$ can be determined with method of progressive selection.

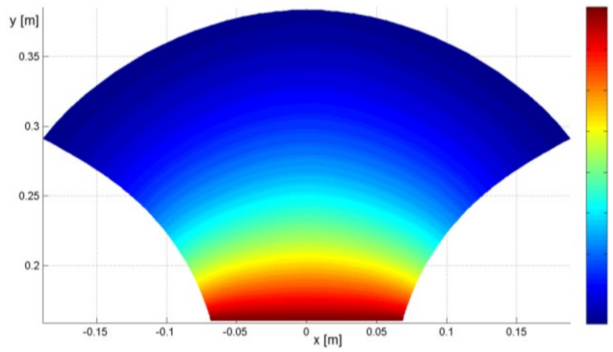

Fig. 2 Meridional velocity field

4. Spatial shape

The plane *y* crosses the meridional plane in the intersection points A and B for the flow rotary symmetric to the plane *x* and limited with the steady surfaces *a* and *b*, see Fig. 1, [1]. It is possible to determine absolute velocity vector **v** in every point of the plane *y*, [1]. The vector **v** is decomposable into circumferential and tangent direction to the junction of points A, B, [1].

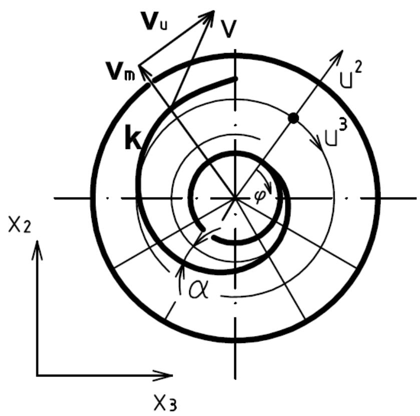

Fig. 3 Plane *x*

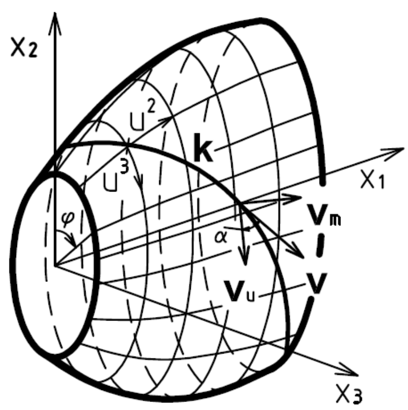

Fig. 4 Plane *y*

Meridians determine streamlines of meridional velocities. Parallel circles are determined with streamlines of absolute velocity vector components in circumferential direction. The streamline of absolute velocity (**v**) vector is represented with curve **k**. This curve determines the characteristic shape of spiral solid on plane *x*. The tangential vector $\mathbf{g}_2$ determines the direction of meridional velocity ($\mathbf{v}_m$). The tangential vector $\mathbf{g}_3$ determines the direction of circumferential velocity ($\mathbf{v}_u$). u^1 is selected to $\mathbf{g}_1 \perp \mathbf{g}_2 \perp \mathbf{g}_3$. Regarding to these relations between surfaces *a* and *b*, the geometric net on the meridional section is interconnected with velocities array. The spiral casing is in accordance with original flow assumption and hydraulic losses are minimized.

The curves u_i^1 of the outlet meridional section of spiral represent the borders of particular meridional sections, see Fig. 1. A fine adjustment of these borders provides possibility to conform to some external design limitations. The spatial rotation of the particular meridional sections creates general spiral spatial shape. The commonly used methods $v_{mean} = const.$ and $rv_u = konst.$ are applied.

The equation of rotation of meridional section for $v_{mean} = const.$ method in curvilinear coordinate system, [1]:

$$\varphi = \frac{2}{\pi} v_{med.} \int_0^1 \int_0^{u^2} \left[|\mathbf{g}_1|^2 |\mathbf{g}_2|^2 - (\mathbf{g}_1 \cdot \mathbf{g}_2)^2 \right]^{\frac{1}{2}} du^1 du^2 \tag{8}$$

The equation of rotation of meridional section for $rv_u = konst.$ method in curvilinear coordinate system, [1]:

$$\varphi = \frac{2\pi Y}{\omega Q \eta_h} \int_0^1 \int_0^{u^2} \left[|\mathbf{g}_1|^2 |\mathbf{g}_2|^2 - (\mathbf{g}_1 \cdot \mathbf{g}_2)^2 \right]^{\frac{1}{2}} \frac{du^1 du^2}{r} \tag{9}$$

5. Volute throat

The circle **c** represents the virtual trajectory of the centre of the last volute throat section. The circle **L** represents trajectory of the point **M**. Its part determines the guiding line of the centre of the volute throat. The circles are tangential. The flow direction changes in the inflexion point **i**. The change of the instantaneous speed of the particle is fluent, thus hydraulic looses are minimized. The centre of the last volute throat section is crossed with the vertical spiral axes. Its coordinates are $[x, y, z] = [0, 0, z_{0L}]$.

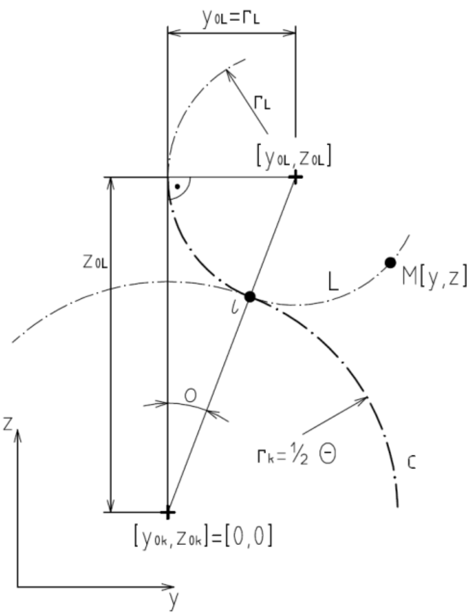

Fig. 5 Guiding line of the centre of the volute throat

The spiral defined by the angle of rotation o is determined with equations:

$$y_{0L} = r_L \tag{10}$$

$$z_{0L} = \frac{r_L}{\tan o} \tag{11}$$

$$r_L = \frac{\Theta}{2\left(\frac{1}{\sin o} - 1\right)} \tag{12}$$

The spiral defined by the height of the outlet pipeline inlet z_{0L} is determined with equations:

$$o = \operatorname{atan}\left(\frac{r_L}{z_{0L}}\right) \qquad (13)$$

$$r_L = \frac{z_{0L}^2}{\Theta} - \frac{\Theta}{4} \qquad (14)$$

The above presented equations represent the common spiral spatial shape. The alternative forms, as tangential spiral, are available through entering of appropriate values of o and z_{0L}. It is also possible to move with the outlet of the volute throat regarding to the spiral solid. The shifts of particular volute throat sections on *x* axes are performed with interpolation of cubic spline. The shifts on *y* axes are performed with addition of appropriate Dy value into (10).

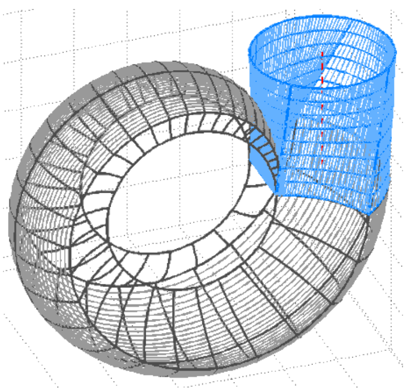

Fig. 6 Result of spiral design performed in software based on the presented mathematical model

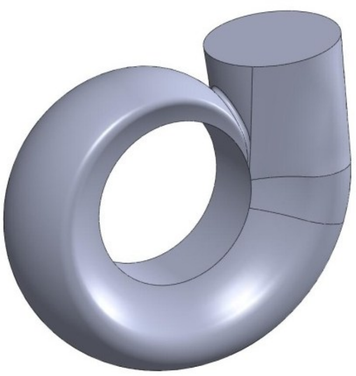

Fig. 7 3D model of fluid within the designed spiral

6. Conclusions

There was introduced the mathematical model of spiral design in this contribution. The design is based on the differential geometry principles in the curvilinear orthogonal net. The fundamental geometric assumptions were performed according to keeping the optimal flow within spiral. The design of outlet meridional section is based on rotational flow (Francis method) assumption. The reason was common distribution of meridional velocity at the centrifugal pump impeller outlet. It is possible to anticipate similar distribution at the spiral inlet. The curves u_i^1 of the outlet meridional section determine other spiral meridional sections. Thus the spiral spatial shape is obtained via rotation of the particular meridional section. The volute throat design was performed in a way that kept fluent instantaneous velocity of particle. The spiral outlet meridional section passes fluently into circular pipeline section within the volute throat. The design results are exportable into one of existing 3D designer programs. The sample of 3D fluid model is presented at the end of the contribution.

7. Acknowledgement

Support of following grant projects is gratefully acknowledged:

The specific research for the year 2011 supported by Faculty of Mechanical Engineering, Brno University of Technology

8. Literature

[1] POCHYLÝ F.: NÁVRH SPIRÁLY ODSTŘEDIVÉHO ČERPADLA, BRNO, BUT FME, 1991

[2] BRDIČKA M., SAMEK L, SOPKO B.: CONTINUUM MECHANICS, ACADEMIA, 2005

[3] SLOUPENSKÝ Z., POCHYLÝ F.: THE MERIDIAN VELOCITY FLOW FIELD WITHIN THE IMPELLER, STU SJF, 2010